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## Wind modelling with nested Markov chains



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### ABSTRACT

Markov chains (MCs) are statistical models used in many applications to model wind speed. Their main feature is the ability to represent both the statistical and temporal characteristics of the modelled wind speed data. However, MCs are not able to capture wind characteristics at high frequencies, and, by definition, in an MC the dependence on events far in the past is lost. This is reflected by a poor match of autocorrelation function of recorded data and artificially generated time series. This study presents a new method for generating artificial wind speed time series. This method is based on nested Markov chains (NMCs), which are an extension of MC models, where each state in the state space can be seen as a self-contained MC. The approach is designed to be flexible, so that the number and distribution of NMC states can be adjusted according to user requirements for model accuracy and computational efficiency. The model is tested on two datasets recorded in two UK locations, one onshore and one offshore. Results indicate that NMCs are able to capture the temporal self-dependence of wind speed data better than MCs, as shown by the better match of the autocorrelation functions of recorded and artificially generated time series.

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### 1. Introduction

The efficient analysis and exploitation of wind energy resources requires models for wind speed at different time scales. The aim of these models is not to forecast the actual wind speed at a certain time, but to generate artificial wind time series that can realistically represent a possible chain of events, i.e. series of wind speeds with a pre-set resolution. Depending on the application, there are some aspects of this “realism” that might be more important than others. This is the case, for instance, of extreme events modelling (Lennard, 2014), or investigation of daily patterns in wind energy production (Scholz et al., 2014), or estimation of total annual energy outputs of wind farms (Hayes and Djokić, 2013b; Hayes et al., 2011, 2012).

Different methods for wind speed modelling have been proposed such as autoregressive moving average (ARMA) models (Kennedy and Rogers, 2003) and Markov chain (MC) models (Jones and Lorenz, 1986). More sophisticated and accurate methods, which may for instance use the knowledge of other quantities such as pressure and temperature, have been developed (e.g. Bitner-Gregersen et al., 2014), but those methods are more

computationally demanding and not suitable for applications where a limited amount of data is available. Despite their simplicity, MCs are able to model the wind time dependence characteristics because they are based on the idea that the probability distribution for the wind at the next time step depends on the current wind state. Other models, such as ARMA, are not able to capture this probability dependence. Therefore, although the need for forecasting has driven academic research to develop better models, the simplicity of MCs makes them a valuable tool as shown by their use in many recent studies. For example, when wind influences a series of decisions that have to be based on current observation, MCs are particularly suited for their property of memory loss (Al-Sabban et al., 2013). Similarly, MCs have been used to model wind turbines when focusing on component failure, that has properties that are independent from the past history (Sunder Selwyn and Kesavan, 2013), or in sailing strategy, where decisions taken at one time step need to be based on the expected wind behaviour at the following time step(s) (Tagliaferri et al., 2014).

However, MCs are not able to capture wind characteristics at high frequencies, but also, by definition, in an MC the dependence on events far in the past is lost. This is reflected in a general good agreement of statistical quantities such as mean and variance, but in a poor modelling of autocorrelation function and power spectral density. A recent study by Brokish and Kirtley (2009) underlines the appeal of MCs for wind modelling in terms of correct

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representation of cumulative distribution function (CDF), but also shows the unsuitability of the model for time steps smaller than 15 min using a convincing example of storage underestimation.

In order to improve the accuracy and the autocorrelation of standard MCs, semi-Markov models have been used, where the time step is not fixed, but it is a random variable that can have any distribution, and the time spent in one state affects the transition probability distribution (D'Amico et al., 2014). In D'Amico et al. (2013), it is shown how semi-Markov processes with memory exhibits a better autocorrelation agreement than conventional MCs. This is due to the ability of this model to keep memory of past transitions through an auxiliary random process representing the moving average of the wind speed.

Some authors (e.g. Shamshad et al., 2005) have also applied second or third order MCs, where time steps are again fixed, and the probability distribution for the next state is dependent not just on the current state, but also on the previous states. Unfortunately, higher order MCs are more computationally demanding, as for instance a third order 32-state MC requires 32,768 state transition probabilities. Therefore, the key advantage of using MCs instead of a more sophisticated method is lost.

In order to improve the MC accuracy and to better model the time correlation at small time steps without an excessive increase of the computational time, we propose the use of nested Markov chains (NMCs) for wind modelling, which is previously considered in the context of “smart grid” analysis in Hayes and Djokic (2013a). With a similar approach to the one presented in D'Amico et al. (2013), we define a model based on MC but with the additional property of keeping a form of memory of past transitions.

The paper is organised as follows: in the Method, we present the principles of MCs and NMCs, how these models are used to forecast the wind speed, and the criteria used to evaluate the results. In the Results, we compare different artificial time series generated with MCs and NMCs with original recorded data. Concluding remarks are summarised in the Conclusions.

## 2. Method

### 2.1. Markov chains

In this section we define MCs and their basic properties. A complete description of MC is out of the scope of this paper and can be found for instance in Norris (1998).

Let  $X_0, X_1, X_2, \dots$  be the stochastic process representing the wind speed. The subscript represents a discrete time step (seconds in this work) and the random variables  $X_i$  can assume values on a discrete set  $S = \{s_1, \dots, s_N\}$ , which is called state space. In the present study, the states  $s_1, s_2, \dots$  are intervals of possible wind speeds, and each interval is identified by its central point. The states are classified in Table 1. With this notation, the wind speed is represented as a time series, or a stochastic process, where, for instance, the events “ $X_0 = s_3$ ” and “ $X_4 = s_8$ ” mean that the wind speed at time 0 (or initial time) is in the interval  $s_3$  and that the wind speed at time  $t = 4$  s is in the interval  $s_8$  respectively. For simplicity, when generating an output time series, we consider just the central point of the interval defining the state. This means that the event “the wind speed value is in the range  $[a, b]$ ” becomes

the event “the wind speed is  $(b + a)/2$ ”. The choice of having wider intervals grouped in the same state for higher wind speeds is justified by the infrequent occurrence of those wind speeds. This results in a trade-off between the number of states and how accurately the higher, more infrequent wind speeds are modelled. However, the occurrences of infrequent wind speeds, and therefore the choice of interval widths, depends on the available dataset (specifically on its length). The Markov property for the process  $\{X_k\}_{k \geq 0}$  asserts that the probability distribution at time  $n$  is dependent on the state at time  $n - 1$ , but independent from what happened before. This property is also referred to as *memory loss*, and is formulated by the following equation:

$$\mathbb{P}\{X_n = s_j | X_{n-1} = s_i, X_{n-2} = s_{i_{n-2}}, \dots, X_0 = s_{i_0}\} = \mathbb{P}\{X_n = s_j | X_{n-1} = s_i\} = p_{ij} \quad (1)$$

where  $s_i, s_j, s_{i_k} \in S$ . Fig. 1 shows a common way of representing MC. The process “jumps” from one state to the next according to the probabilities associated to the arrows. It is clear from the representation that the transition probabilities depend on the current state, but not on the previous ones. The transition probabilities are naturally represented in a transition matrix  $P = \{p_{ij}\}$ , where the elements of the matrix,  $p_{ij}$ , are the probabilities defined in Eq. (1). The  $i$ th row of the matrix  $P$  represents the discrete probability distribution for the next state when the current state is  $i$ . The probability distribution for the initial state  $X_0$ , or *initial distribution* is conventionally represented as a column vector  $P_0$ , where the element  $p_i^0$  is defined in Eq. (2), or the initial state could be arbitrarily selected to start the process (for instance, as the mode or median value from the dataset):

$$p_i^0 = \mathbb{P}\{X_0 = s_i\} \quad (2)$$

State space, transition matrix and initial distributions uniquely define an MC process.

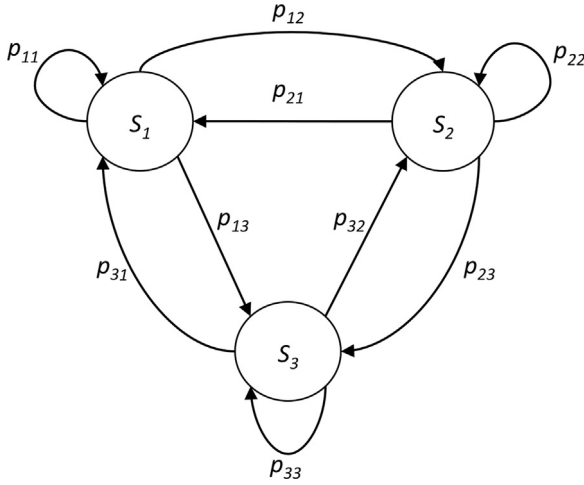
### 2.2. Nested Markov chains

In the NMC approach, the wind series is built using an auxiliary MC. Let  $T$  and  $t$  be two different time steps, where  $T$  is a multiple of  $t$ . For instance, throughout this paper  $t = 1$  s. Let  $S = s_1, \dots, s_N$  be a finite state space. We define  $Y_0, Y_1, \dots$  the MC on the space state  $S$ , representing the average wind speed over a period of length  $T$  with transition matrix  $P$ . In the following, this process will be referred to as the *outer* MC. We generate a sequences of wind speed time series of duration  $T$  with time step  $t$  using the transition matrix  $P^{Y_i}$ , where the element  $p_{ij}^{Y_i}$  represents the probability that the wind at instant  $n$  is in state  $s_j$  given the event that at time  $n - 1$  it was in state  $s_i$  when the average over the period  $T$  is  $s_k$ . Those models will be referred to as *inner* MC. The output process is the sequence of realisations of the inner MC, i.e. one series if inner states with step  $t$  for each state of the outer MC. Fig. 2 shows a graphical representation of the relationship between inner and outer MC.

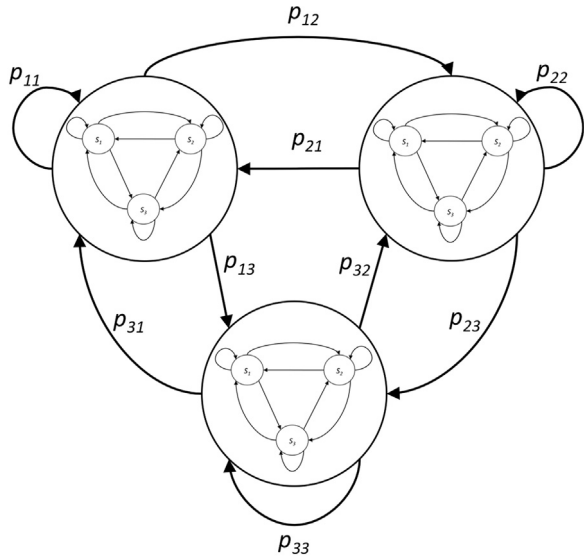
The output process now does not strictly follow the Markov property, because the probability distribution for time  $n$  does not depend only on the state at time  $n - 1$ , but also on what happened in the previous hour. However, if we consider the process within one hour, this is an MC. Also the outer process is an MC. In fact, an

**Table 1**  
State spaces.

State	$s_1$	$s_2$	...	$s_{26}$	$s_{27}$	$s_{28}$	$s_{29}$	$s_{30}$	$s_{31}$	$s_{32}$
Interval (m/s)	0–1	1–2	...	25–26	26–28	28–31	31–34	34–39	39–43	43–54
Output (m/s)	0.5	1.5	...	25.5	27	29.5	32.5	36.5	41	48.5



**Fig. 1.** Representation of a Markov chain with three states  $S_1$ ,  $S_2$ ,  $S_3$ , with transition probabilities  $p_{ij}$ .



**Fig. 2.** Representation of a nested Markov chain with three states, where the probabilities  $p_{ij}$  are associated to the transitions of the outer chain.

NMC can be seen as an extended MC, in which each state is a self-contained stochastic MC sub-process.

In this study, the state space  $S$  is the one defined in Table 1 for both the outer and all the inner processes. This is not the only possible choice, and other options could involve two different state spaces depending on what the outer process is modelling. For instance, it could involve the characterisation of other weather components, such as occurrence of rain and clouds, so that the inner MCs correspond to different MCs for sunny/cloudy/rainy days. The process can be generalised even more, using non-Markovian models for the generation of the inner time series. For instance, the inner MC may be replaced with an AR mode, and the model would become a Markov switching AR model (Ailliot and Monbet, 2012; Pinson and Madsen, 2012).

### 2.3. Artificial time series

In this section we describe the process for computing the initial distribution and the transition matrix that uniquely defines the MC model starting from recorded data, and then how a new artificial time series can be generated from those transition matrices

via a Monte Carlo simulation. We first describe the procedure for a generic MC.

The first step is to define the state space, shown in Section 2.1 (see Table 1).

The initial distribution corresponds to the empirical distribution function for the entire wind series. It is computed by dividing the dataset into bins corresponding to the state space intervals, and normalising the vector of the occurrences in every bin. For instance, to compute  $p_1^0 = \mathbb{P}\{X_0 = s_1\}$ , which is the probability that the first element of the wind time series is in the interval  $s_1$ , we count the number of times that there is a value belonging to the interval  $s_1$  in the entire recorded time series and divide it by the total number of recorded values.

The transition matrix is obtained in a similar way. The generic element  $p_{ij}$  of the transition matrix is computed by counting how many times a value in the interval  $s_i$  is followed by one in the interval  $s_j$  in the recorded wind speed time series, normalised over the number of occurrences of values in bin  $s_i$ . Formally, this means using a maximum likelihood estimator for the transition probabilities. Once the probability distributions have been computed, it is possible to generate wind speed time series of arbitrary length through a Monte Carlo simulation. This can be achieved by using a conventional random number generator. We generate a series of independent identically distributed random variables  $z_0, z_1, z_2, \dots$  uniform on the interval  $[0, 1]$ . Let  $P_0 = [p_1^0, \dots, p_N^0]$  be the probability vector representing the initial distribution. The initial state  $X_0$  is defined as  $X_0 = s_i$  if

$$\begin{cases} \sum_{k=0}^i p_k^0 < z_0 \\ \sum_{k=0}^{i+1} p_k^0 > z_0 \end{cases} \quad (3)$$

Similarly, for the subsequent time steps, the value for the general  $X_t$ , given the previous state  $X_{t-1} = s_i$  is defined to be  $X_t = s_j$  if

$$\begin{cases} \sum_{k=0}^j p_{ik} < z_i \\ \sum_{k=0}^{j+1} p_{ik} > z_i \end{cases} \quad (4)$$

The described procedure, which can be used to generate an artificial time series based on an MC model, can be extended to NMC as follows. The initial distribution is computed as described in Eq. (3). The dataset is processed to generate an auxiliary dataset constituted by hourly average values. This auxiliary dataset is used to compute the transition matrix for the outer process. The original dataset is also subdivided in  $N$  smaller datasets, where sequences of 3600 values belong to the  $i$ th set if their average corresponds to the state  $s_i$ . Each of these dataset is used to generate a transition matrix  $P_i^s$ . For generating the NMC-based time series, the outer MC time series is generated first, by using the procedure described above for conventional MC models. Then, for each outer state, another MC of exactly 3600 time steps is generated. In this case, attention should be paid to the initial state of each of those MCs. In fact, the last state of the previous MC is used as initial state for the following one.

The value for the general  $X_t$ , given the previous state  $X_{t-1} = s_i$  and given the outer state  $Y_t = s_i$  is defined to be  $X_t = s_j$  if

$$\begin{cases} \sum_{k=0}^j p_{ik}^l < z_i \\ \sum_{k=0}^{j+1} p_{ik}^l > z_i \end{cases} \quad (5)$$

The choice of the state space is highly dependent on the application. The space used for a 32-states MC in this study is again the one defined in Table 1. In order to investigate the sensitivity of model accuracy and computational requirements on the number

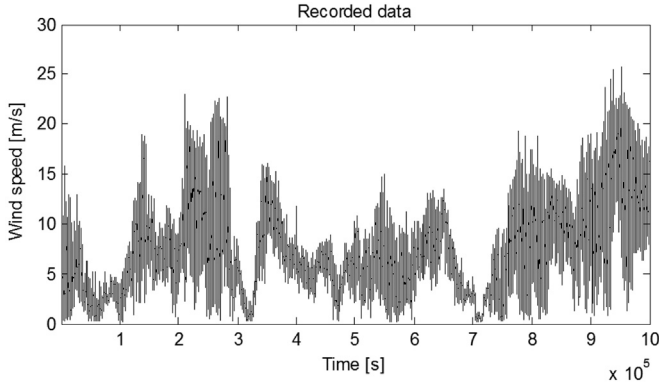


Fig. 3. 1,000,000-s example of recorded data.

of states, we model also MCs with just 16 and 8 states. The states for those other models are defined by merging close states. The optimal number of states depends on the application and on the amount of available data.

It is also important to note that the outer state does not strictly represent the wind speed average over one hour. In fact, for each hour a certain average (corresponding to a state  $s_i$ ) is assumed. This value  $s_i$  identifies a transition matrix which is then used to generate 3600 values. As this is a finite number of states, the actual average of the 3600 values generated may be, and typically is, different from  $s_i$ . The outer state can rather be interpreted as an *expected average*. In fact, due to convergence laws for MC (Norris, 1998), if the inner process had an infinite number of points, its average would converge to the expected average.

#### 2.4. Model evaluation

The proposed method is first evaluated by testing it on a recorded onshore dataset, which is a high-resolution wind speed dataset recorded at a site located on the west coast of Scotland (Anderson, 2006). Wind speed at 1 Hz resolution was measured over a period of around 150 days. Fig. 3 shows an example of recorded data. An additional dataset recorded at an offshore site is discussed in Section 3.

The performance of the proposed NMC with different number of states (8, 16 and 32) is compared with a standard MC model with 32 states. In order to provide a comparison with a standard procedure in time series analysis, the results achieved with an autoregressive moving average model (ARMA) are also presented.

An ARMA process of parameters  $p$  and  $q$  is defined by the following equation:

$$X_t = \sum_{i=1}^p a_i X_{t-i} + \sum_{i=0}^q b_i \epsilon_{t-i} + c \quad (6)$$

where  $a_i$ ,  $b_i$  and  $c$  are the parameters defining the model, and  $\epsilon_i$  are white noise error terms (Bendat and Piersol, 2011).

For each of these methods we compute mean, variance, coefficient of determination  $R^2$  and autocorrelation. Given a time series  $X_0, X_1, \dots, X_M$ , the autocorrelation  $r_k$  is defined in the following equation:

$$r_k = \frac{\sum_{m=1}^{M-k} (y_m - \bar{y})(y_{m+k} - \bar{y})}{\sum_{m=1}^M (y_m - \bar{y})^2} \quad (7)$$

where  $M$  is the total number of data points in the time series,  $y_m$  the series value at time point  $m$ ,  $k$  is the number of lags, and  $\bar{y}$  is the overall average wind speed value.

The best values for  $p$  and  $q$  are chosen by using the Bayesian

Table 2  
Statistics for the different  $T$ .

$T$ (s)	Mean (m/s)	Standard deviation	$R^2$
1	8.30	4.71	0.843
10	8.95	4.32	0.912
1000	9.74	3.28	0.989
3600	9.71	3.21	0.991
10,000	9.16	3.45	0.993
100,000	8.15	4.21	0.821
1,000,000	7.12	3.23	0.854
Recorded	9.45	4.88	–

Table 3  
Statistics for the recorded 150-day dataset and for the different models tested.

Model	Mean (m/s)	Standard deviation	$R^2$
Recorded	9.45	4.88	–
NMC-32	9.71	3.21	0.991
MC-32	8.30	4.71	0.843
ARMA	6.91	2.92	0.754

information criterion (BIC). This is based on an exhaustive search, simulating different ARMA models with different values for  $p$  and  $q$ . For each model, the loglikelihood objective function is computed. This function is then used to compute the BIC measure of fit. Further details on ARMA processes and BIC can be found, for instance, in Stoica and Selen (2004).

### 3. Results

In this section we present and analyse different artificial time series, compared with original recorded data. The artificial time series are generated using

1. A 32-states NMC.
2. A 32-state MC.
3. An autoregressive moving average model, ARMA( $p, q$ ) with the optimal values  $p=8, q=8$ .

The first investigation is carried out to identify the optimal  $T$ , representing the permanence time in an outer state. Different 32-states NMC models are tested and the results are summarised in Table 2. The best fit is obtained for values of  $T$  between 1000 and 10 000 s.

The NMC in the following are based on  $T=3600$  s, i.e. the outer process is based on hourly averages. In fact, 3600 s lies in the optimal range which yields the highest correlation values. The statistical properties of the original dataset and the artificial time

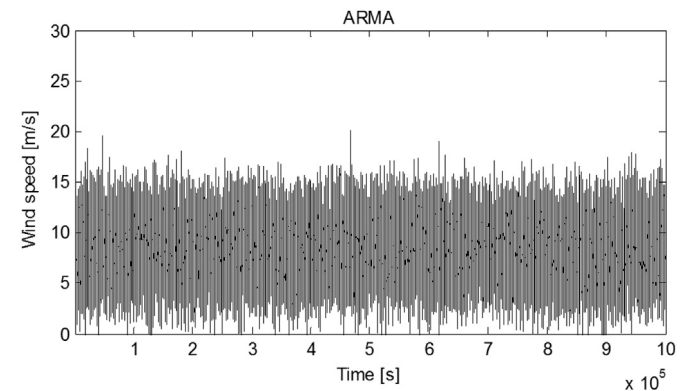


Fig. 4. Example of 1,000,000-s time series generated using an ARMA(8,8) model.



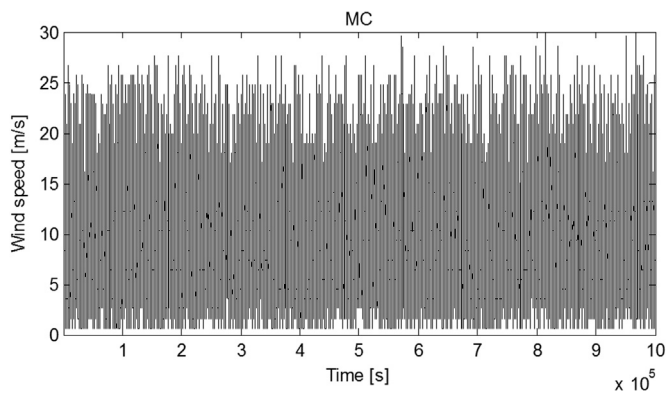


Fig. 5. Example of 1,000,000-s time series generated using an MC model with 32 states.

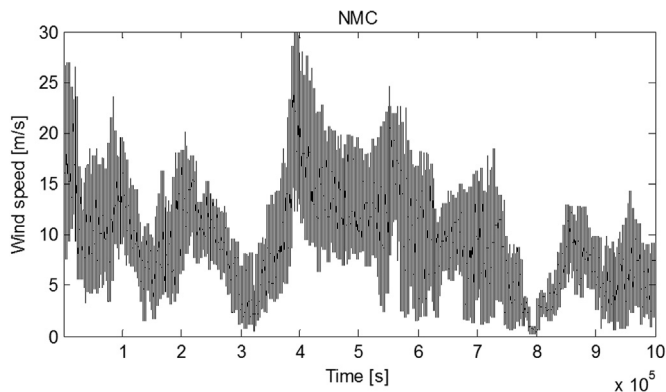


Fig. 6. Example of 1,000,000-s time series generated using an NMC model with 32 states.

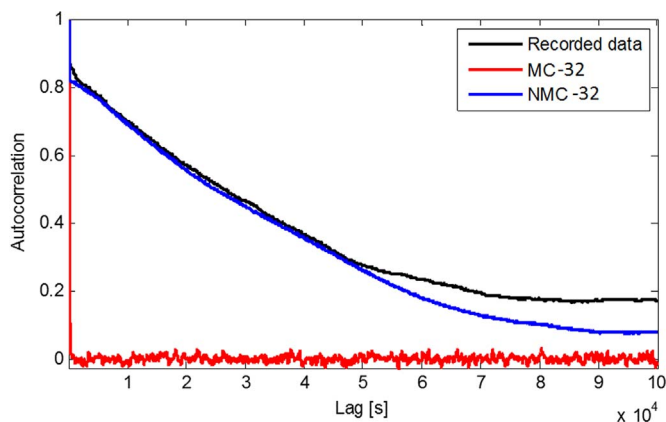


Fig. 7. Autocorrelation plots for recorded data and for MC and NMC models with 32 states.

series are summarised in Table 3. The ARMA model is the one that differs the most from the original data and this is reflected by all the statistical indices. Although the MC model has a standard deviation that is closer to the one of the recorded data, the match of the CDF, represented in the value  $R^2$ , is improved by the use of the NMC. In general, all the  $R^2$  values are high, but a close match of the CDF is, however, expected from the construction of the model and the use of the maximum likelihood estimator.

Fig. 4 shows a 1,000,000-s (278-h) example of a time series generated using the ARMA model. Using the BIC, the best  $p$  and  $q$  were found to be 8 and 8, respectively. A qualitative comparison with the recorded data shows higher fluctuations in the wind speed values, although there is a slightly higher number of values

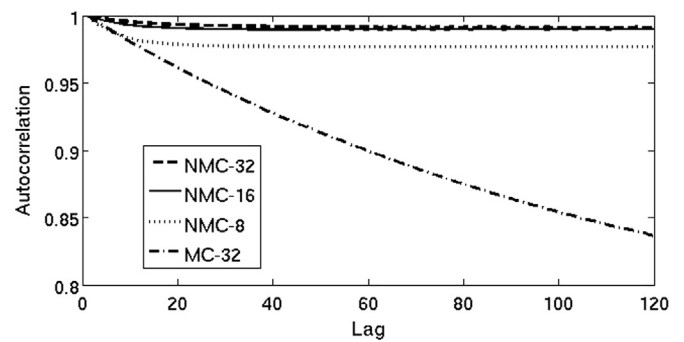


Fig. 8. Autocorrelation plots for recorded data, 32-state MC and three different NMC models.

around the mean. The low  $R^2$  value reflects this problem.

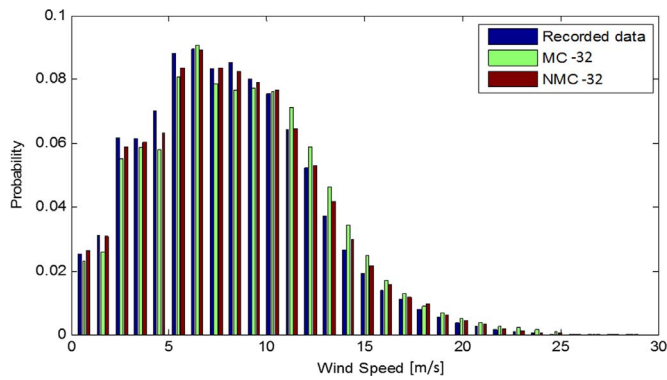
Fig. 5 shows a 1,000,000-s example of a time series generated using the 32-states MC model, while Fig. 6 shows a 1,000,000-s example of time series generated using the 32-states NMC. The two time series are qualitatively very different, although the statistics presented in Table 3 have close values. In particular, a value that can be misleading is the standard deviation. The standard deviation values are very similar because they are computed over the long output time series (150 days), and their similarity is again a consequence of the closeness of CDF. When computed over smaller intervals, the variance for the NMC model is, on average, closer to the one of the original data than the one for MC. In fact the occurrences of different values along the whole time series are similar. However, when using NMC, similar values are “clustered” together around a fixed hourly average.

The quantitative improvement of the proposed NMC over MC in terms of autocorrelation is shown in Fig. 7. The NMC model allows us to preserve the autocorrelation significantly longer than the MC model, which deviates from the original data after approximately 100 time steps. Fig. 8 shows the autocorrelation for NMC models with 8, 16 and 32 states in comparison with a 32-state MC model, focusing on the differences over the first lags. The results show that when more states are used for the NMC model, the autocorrelation improves. However the biggest gain is obtained when going from 8 to 16 states, while going from 16 to 32 states brings a marginal improvement. Even with just 8 states, an NMC-based model allows a significant improvement in the autocorrelation modelling with respect to a conventional MC model.

The increased similarity of the autocorrelation function to the one of the original data means that, overall, the NMC model can be used to generate more realistic wind speed time series when compared to a conventional MC model. This becomes particularly important when the artificially generated series are used as input for computations in certain applications. In fact, if the computations depend not only on the current state, but also on the previous states, then it is important that not only the single values are realistic (as captured, for instance, by the PDF), but also that the relationship between consecutive value is respected, and this is exactly what is captured by the autocorrelation function. This is crucial, for instance, in presence of hysteresis. An example is the pitch control of the turbines' blade, where there could be

Table 4  
Comparison of computational times.

Model	1 hour output (s)	1 year output
MC-32	0.13	30 min, 39 s
NMC-8	0.18	26 min, 16 s
NMC-16	0.44	1 h, 4 min, 21 s
NMC-32	0.95	2 h, 18 min, 48 s



**Fig. 9.** Empirical probability density function for recorded data and for MC and NMC models with 32 states.

**Table 5**

Statistics for the verification dataset and for the different models tested.

Model	Mean (m/s)	Standard deviation	$R^2$
Recorded	10.12	5.15	–
NMC-32	10.06	4.73	0.991
MC-32	10.54	4.71	0.913
ARMA	9.78	4.38	0.824

hysteresis in the case of dynamic stall (Taylor et al., 2015). Other cases include control energy storage systems, where the optimal operation depend not only on the energy currently produced, but also on the previously produced energy (Setas Lopes et al., 2016). Applications beyond the wind energy field include decision-making in competitive sailing, where the optimal decisions should be taken according to the wind observed over a certain time window rather than a single time-step. The improvement in NMC could allow routing algorithms, such as the one presented by Tagliaferri et al. (2014), to include multi-step computations.

In order to quantify the trade-off between accuracy of the model and complexity, Table 4 shows the computational times required by the model to generate: (i) one hour of 1 Hz wind speed output ( $60 \times 60 = 3600$  data points) and (ii) one year of 1 Hz wind speeds ( $365 \times 24 \times 60 \times 60 = 31,536,000$  data points). All analyses were carried out in Matlab using a 2.27 GHz Intel micro-processor. The computation times are a function of the number of allowable state transitions in the model, i.e. reducing the number of NMC states results in shorter computation times.

The choice of the optimal number of states generally depends on a number of factors related to the target application. For instance, some studies such as the one focussing on the comparison of two energy storage options in Hayes et al. (2016), require long time histories (in that case 40 years) with short time steps. Assuming the same computational power as the one used for this study, and the same time step of 1 Hz (although the study cited uses a longer time step) generating this time history would require almost four days of computation using the NMC-32, and less than one day using NMC-8. However, this type of analysis is usually carried out at a planning stage (i.e. off-line), as part of the longer-duration feasibility studies and design processes, which involve a number of options, parameters and factors, but are not time-critical. Accordingly, the additional computation time required for increasing the quality of the generated dataset will not necessarily have an impact on the overall time needed for the study. In other cases however, for instance when using probabilistic models for forecasting wind speed (Carpinone et al., 2015), the computations need to be carried out in real-time, but these applications usually require shorter time series. Considering a one-hour time series, the difference between using NMC-8 and NMC-32 is in most cases

negligible. Nevertheless, if the available computational resources are limited, or if the processing requirements for the application are high, a lower number of states could still be chosen for some real-time implementations, for instance in sailing applications as described in Tagliaferri et al. (2014).

Fig. 9 shows a comparison of the empirical probability density functions of the original data with the empirical PDFs of the MC and NMC model. Both models are in good agreement with the data, as in both cases the parameters were calculated using maximum likelihood estimators. However, and crucially for the presented analysis, the MC model leads to the higher probability values for low-probability events (the right tail of the curve), which means a much higher occurrence and therefore less realistic representation of extreme values. In fact, as shown in Dobakhshari and Fotuhi-Firuzabad (2009), when studying wind energy applications it is important in wind energy to take into account the so-called “cut-in” and “cut-out” wind speeds. These two values represent the boundaries of the range of wind speed at which energy is produced by wind turbines: if the wind speed is either lower than the cut-in speed, or higher than the cut-out speed, the output power of a wind turbine is zero. Therefore, for reliable analysis, it is important that the model used for generating wind speed time series assigns adequate probability to wind speeds outside this range.

In order to further demonstrate application of the presented NMC methodology, the calculations carried out for the onshore dataset are repeated by using an additional dataset from an off-shore site (Noordzee Wind, 2013). Table 5 shows a further comparison between the different models, using this alternative dataset. The results are in agreement with those presented for the onshore dataset and improvement in autocorrelation function is also confirmed. However, for this particular case the optimal ARMA model was found to be for  $p=7$ ,  $q=6$ .

#### 4. Conclusions

Markov chains constitute a useful and easy tool for generating synthetic artificial wind speed time series with a low computational cost. Compared to other common models, they are easy to implement, have low computational requirements and are able to represent correctly the first order statistics of recorded data. However, due to their “loss of memory”, these models are not able to capture the time dependency on past values, and this is typically shown in a poor agreement of the autocorrelation function.

In this study, we have proposed for the first time the use of nested Markov chains to model wind speed. In a conventional MC model, the probability distribution for each value depends only on the previous one. In an NMC-based model, it depends also on the hourly average and on the wind speed in the past hour, taking into account lower frequencies of the time series. We tested the model by generating one-second-step wind speed time series with MC and NMC approach for one onshore and one offshore site, where the NMC takes into account the hourly averages of the wind speed. We show that with NMC it is possible to get a significant improvement in the autocorrelation function for the artificially generated time series. This means that NMC can better model the temporal self-dependence of analysed time series than a conventional MC.

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